

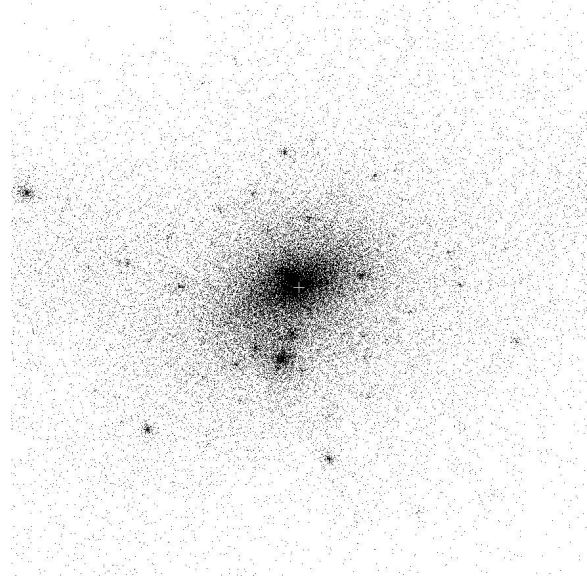
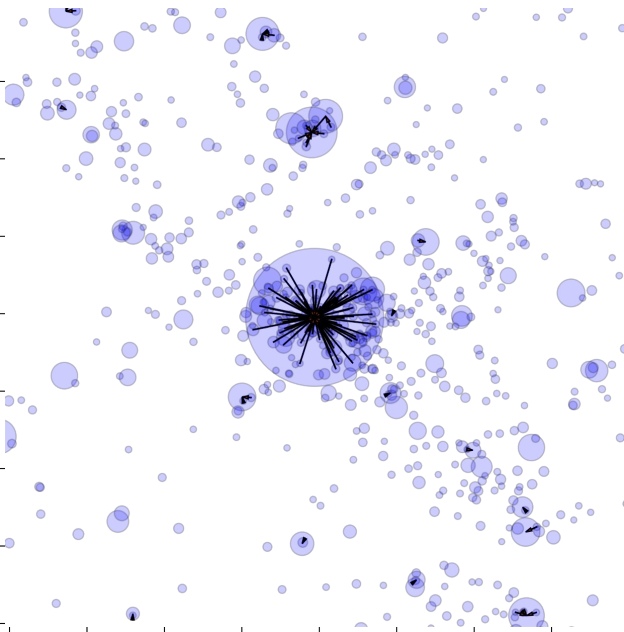
- [2305.16176](#) [astro-ph.CO]
- [2206.05578](#) [astro-ph.CO]
- [2306.08028](#) [astro-ph.CO]
- JCAP 09 (2022) 077
- Astrophys.J. 949 (2023) 2, 67

Analytic kernels for modeling the topological features and self-interactions in dark matter halos

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September 2023

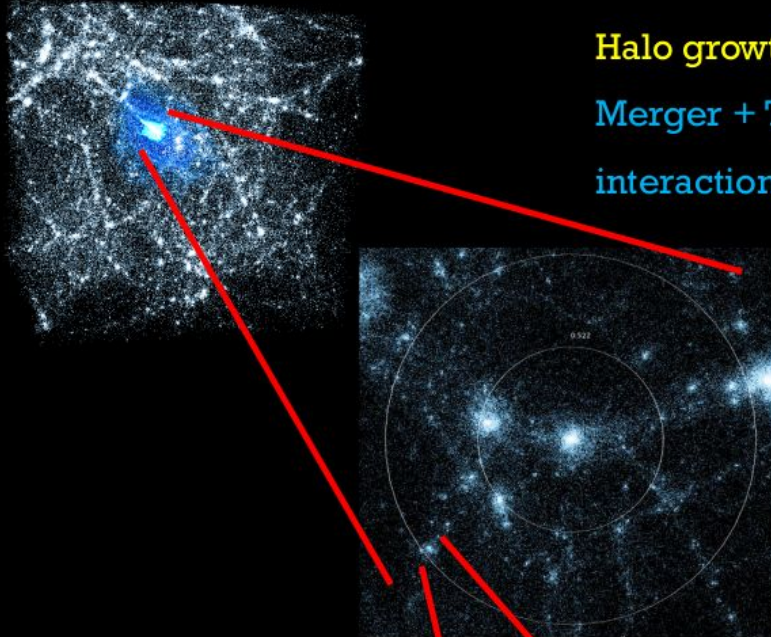
Recent works with
Hai-Bo Yu (UCR),
Ethan O. Nadler (Carnegie OBSY & USC)
Yi-Ming Zhong (UChicago & CityU HK)



Structure formation is hierarchical

Large scales

Cold Dark Matter



Halo growth

Merger + Tidal interactions

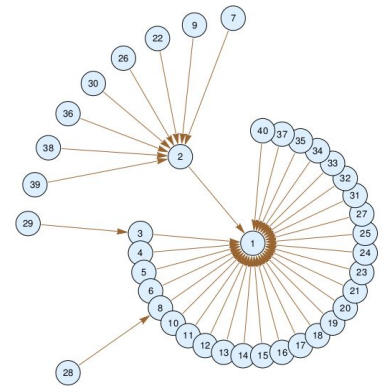
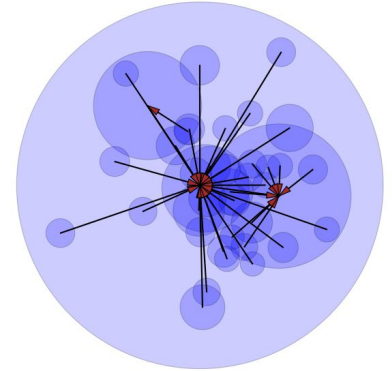
Small scales

Virialize

+ Could be self-interacting



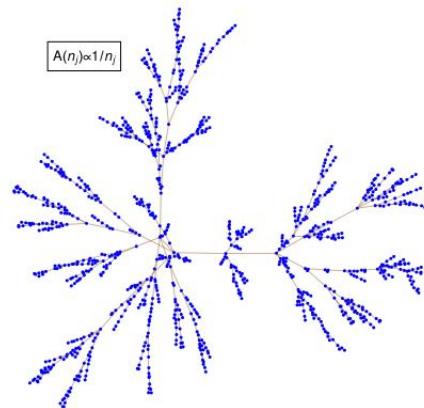
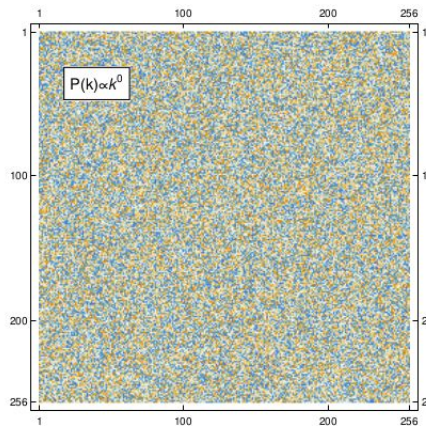
ΛCDM is well established, but there is a different way to look at it



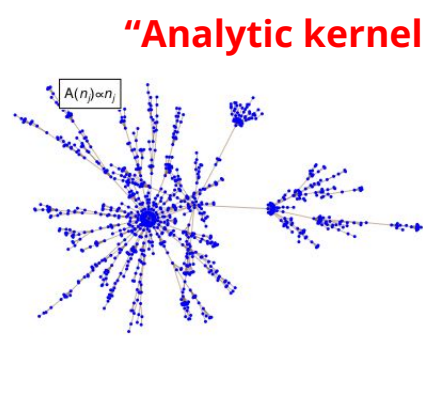
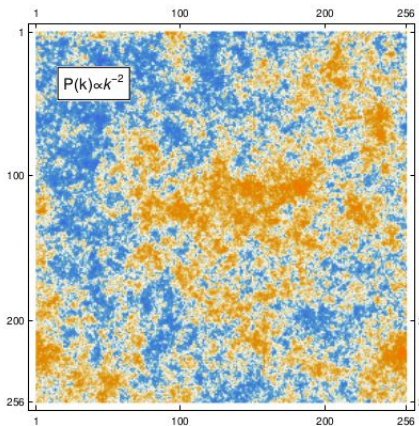
Network theory
can offer a new
perspective of
looking at
structure
formation

k : node degree, i.e.,
number of links
connected to a
node

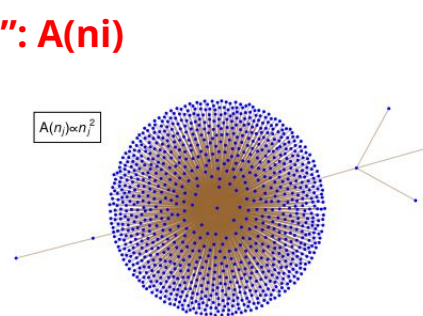
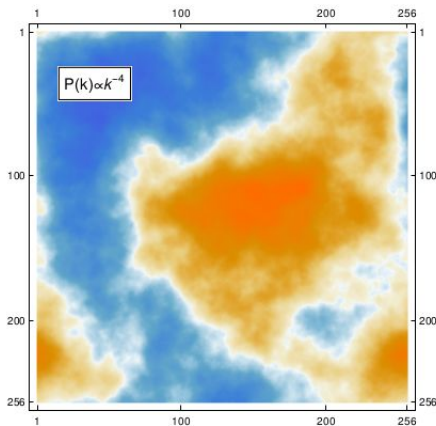
$$\text{Attachment Probability} = \frac{A_i}{\sum_j A_j}$$



White noise



Scale free



Winner takes all

“Analytic kernels”: $A(n_i)$

Perspectives with graphs

Symmetries

- Power indices of matter power spectra
- Scale symmetry and preferential attachment

Physical processes

- Mergers
- Tidal stripping
- WDM/Feedback /SIDM?

A natural structure for capturing complicated correlations: Graph Neural Network

Around 10 GNN papers in astrophysics appeared in the past 1.5 years

Scale symmetry and preferential attachment (PA)

Linear attachment kernel (BA model)

- * Simple realization of PA:
 - Early attachment advantage
- * power law degree distribution: k^{-3}

Breaking the symmetry softly

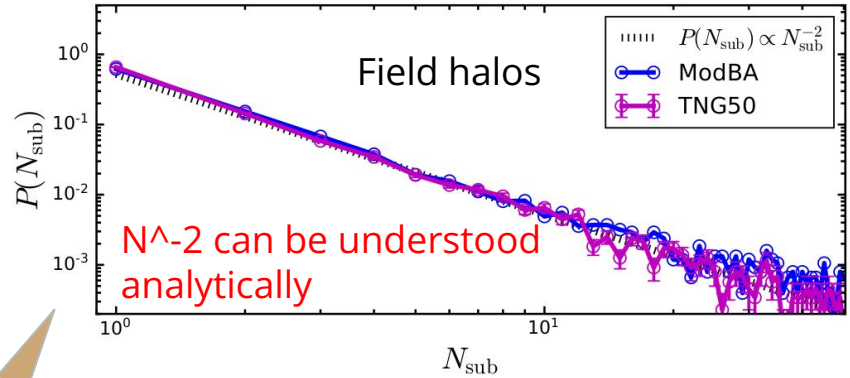
- * Major mergers: **(Analytic kernels)**

$$A_{j,1} = \alpha k_j^{\beta+1} / (k_j + \alpha k_j^{\beta}) \xrightarrow{\text{large } k_j} k_j$$

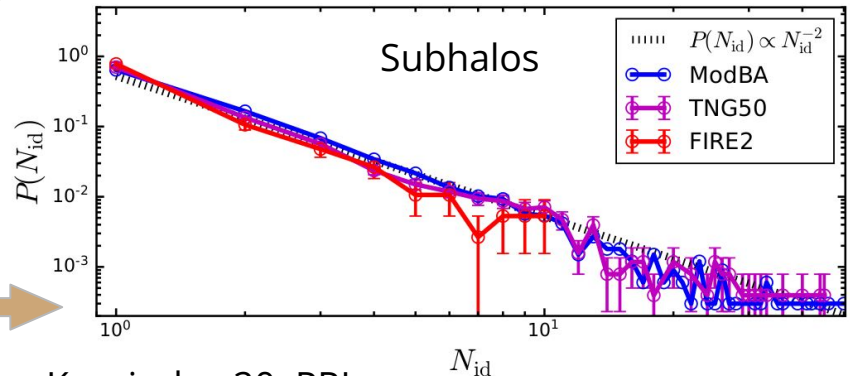
- * Tidal evolution:

$$A_{i,2} = k_i + \alpha k_i^{\gamma} \xrightarrow{\text{small } k_j} k_j$$

- * Power index = $-(3 + \sqrt{1 + 8\alpha'})/2 \rightarrow -2$ ($\alpha' \rightarrow 0$)



Yang & Yu 2206.05578



Krapivsky+20, PRL

Topological information from graph metrics

Topological information

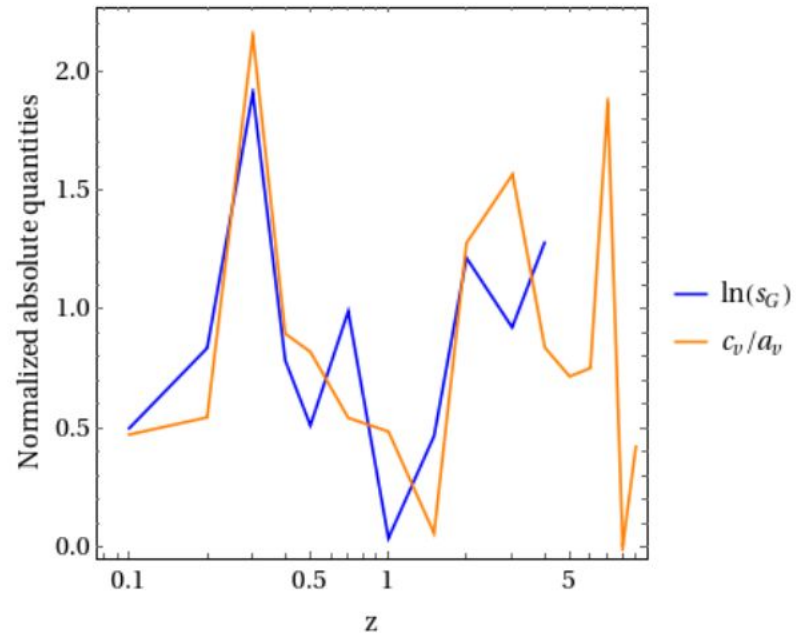
- Length of edges are not measured
- Self-similarity can be measured
- Morphological information beyond ellipsoidal parameters

Given a degree sequence $D=\{k_1, k_2, \dots, k_N\}$, one can construct a graph maximizing s_G

(Li Lun, et al 2006)

- $s_G / s_{max} = 0.98$ (**FIRE2 simulations**)
- $s_G / s_{max} = 0.93$ (**Model constructions**)

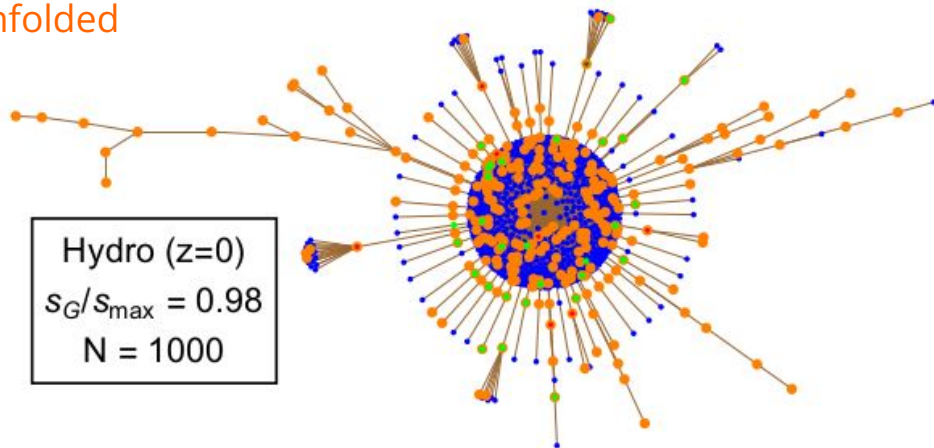
The most massive host in TNG-50-1



A follow-up paper in preparation

An efficient structure for gathering characteristic features

Rich inner structure
unfolded

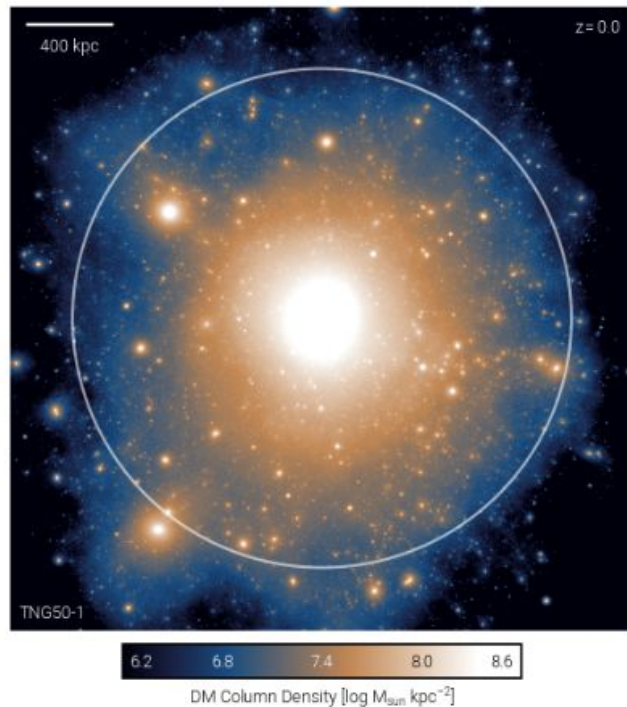


Orange: enriched stellar mass

Green: enriched gas mass

Magenta: hosting massive black holes

The most massive host in TNG-50-1



A follow-up paper in preparation

$$\rho_{\text{SIDM}}(r) = \frac{\rho_s}{\frac{(r^\beta + r_c^\beta)^{1/\beta}}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

$\beta=4$

$$\frac{\rho_s}{\rho_{s,0}} = 2.033 + 0.7381\tilde{t} + 7.264\tilde{t}^5 - 12.73\tilde{t}^7 + 9.915\tilde{t}^9 - (1 - 2.033)(\ln 0.001)^{-1} \ln(\tilde{t} + 0.001),$$

$$\frac{r_s}{r_{s,0}} = 0.7178 - 0.1026\tilde{t} + 0.2474\tilde{t}^2 - 0.4079\tilde{t}^3 - (1 - 0.7178)(\ln 0.001)^{-1} \ln(\tilde{t} + 0.001),$$

$$\frac{r_c}{r_{s,0}} = 2.555\sqrt{\tilde{t}} - 3.632\tilde{t} + 2.131\tilde{t}^2 - 1.415\tilde{t}^3 + 0.4683\tilde{t}^4,$$

$$\tilde{t} \equiv t/t_c$$

Analytic kernels for self-interacting dark matter (SIDM) halos

Parametric model: 2305.16176 [astro-ph.CO]

Universal gravothermal evolution

Time reversal

$\mathbf{x} \rightarrow \mathbf{x}$

$t \rightarrow -t$

$\mathbf{v} \rightarrow -\mathbf{v}$

$$\frac{3\rho}{2m} \left(\frac{\partial T}{\partial t} + \langle v_i \rangle \nabla_i T \right) = -\boxed{\nabla_i J_i} - P \nabla_i \langle v_i \rangle - \boxed{\Pi_{ij}^{\text{vis}} \partial_i \langle v_j \rangle} - \rho \nabla_i \Phi \cdot \langle v_i \rangle$$

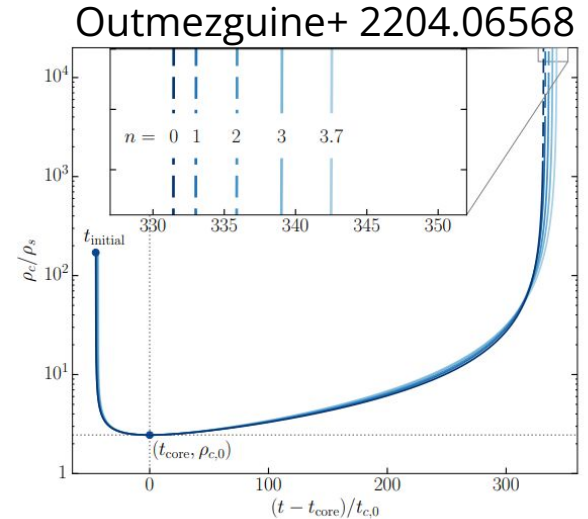
Heat conduction breaks time reversal invariance

Arrow of time dependent on SIDM

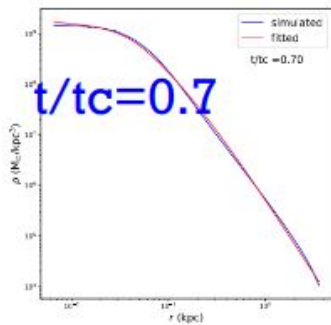
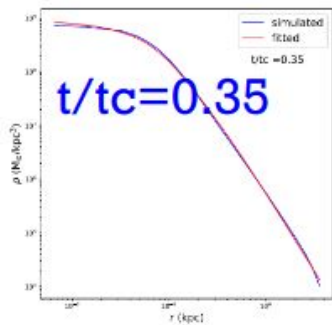
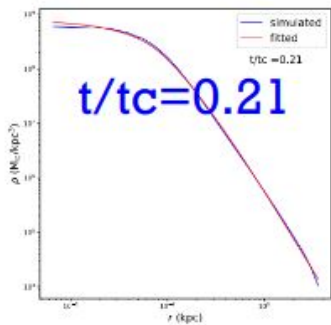
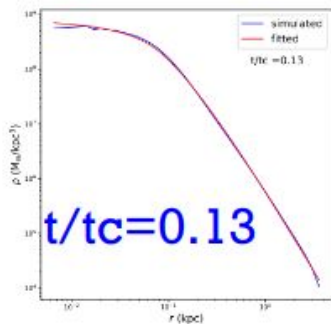
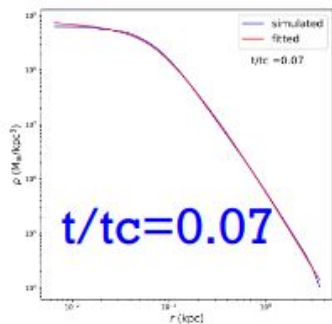
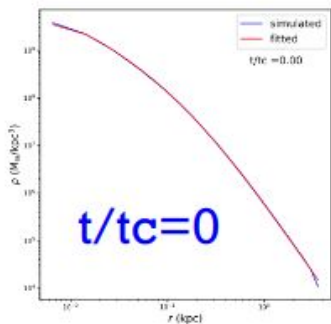
Could we absorb SIDM effect into the **arrow of time**? When conduction \sim # of scatterings, **Yes!**

$$\tilde{t} \equiv t/t_c \quad t_c = \frac{150}{C} \frac{1}{(\sigma_{\text{eff}}/m)\rho_{\text{eff}}r_{\text{eff}}} \frac{1}{\sqrt{4\pi G\rho_{\text{eff}}}}$$

In long-mean-free-path regime, $\kappa \sim \sigma$ and the fluid equations can be put in a universal form (Zhong+2306.08028)



A universal parametrization of the density profile under SIDM



$$\rho_{\text{SIDM}}(r) = \frac{\rho_s}{\frac{(r^\beta + r_c^\beta)^{1/\beta}}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$$

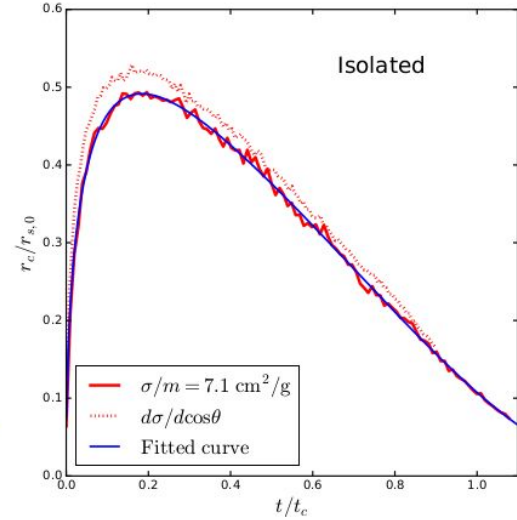
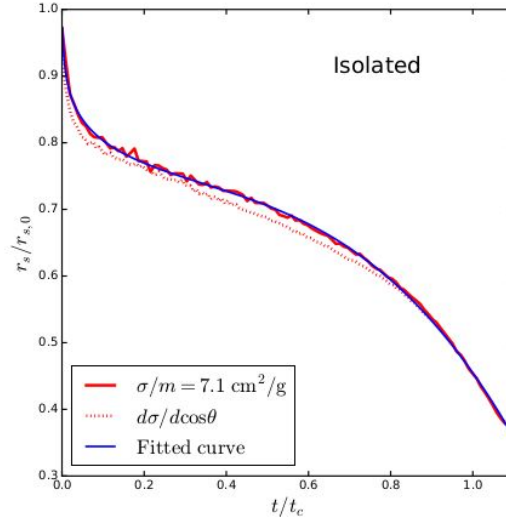
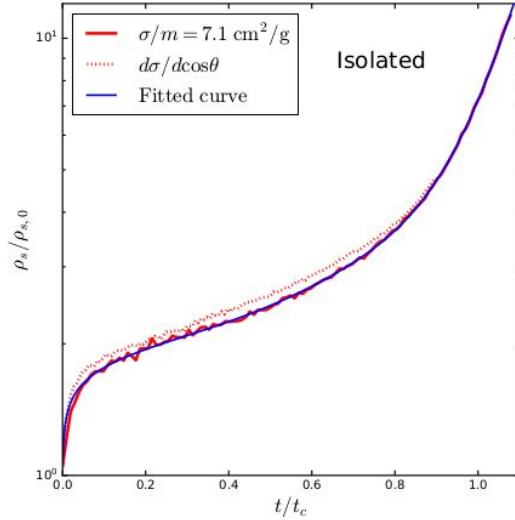
$\beta=4$

Simulated

Fitted

Similar performance found for the "Read" profile, will appear in our updated paper soon

Analytic kernels: Trajectories of the profile parameters



$$\frac{\rho_s}{\rho_{s,0}} = 2.033 + 0.7381\tilde{t} + 7.264\tilde{t}^5 - 12.73\tilde{t}^7 + 9.915\tilde{t}^9 - (1 - 2.033)(\ln 0.001)^{-1} \ln(\tilde{t} + 0.001),$$

$$\frac{r_s}{r_{s,0}} = 0.7178 - 0.1026\tilde{t} + 0.2474\tilde{t}^2 - 0.4079\tilde{t}^3 - (1 - 0.7178)(\ln 0.001)^{-1} \ln(\tilde{t} + 0.001),$$

$$\frac{r_c}{r_{s,0}} = 2.555\sqrt{\tilde{t}} - 3.632\tilde{t} + 2.131\tilde{t}^2 - 1.415\tilde{t}^3 + 0.4683\tilde{t}^4,$$

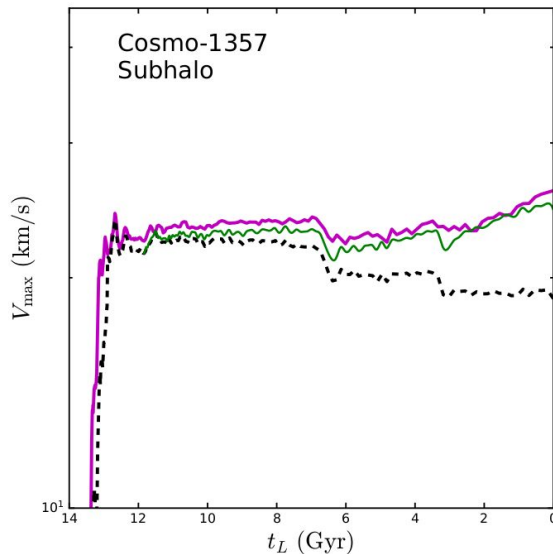
$$\tilde{t} \equiv t/t_c$$

An integral approach of applying the kernels

Tidal radius introduced for subhalos

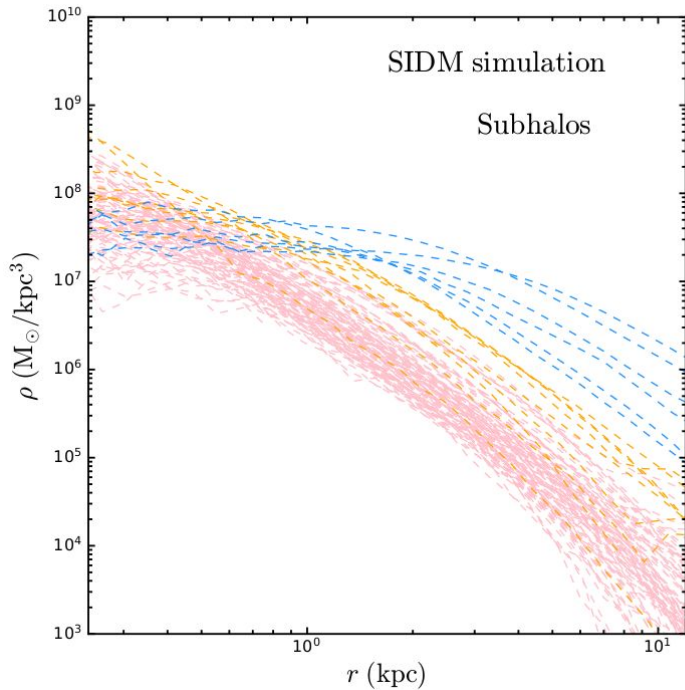
$$V_{\max}(t) = V_{\max,\text{CDM}}(t_f) + \int_{t_f}^t dt' \frac{dV_{\max,\text{CDM}}(t')}{dt'} + \int_{t_f}^t \frac{dt'}{t_c(t')} \frac{dV_{\max,\text{Model}}(\tilde{t}')}{d\tilde{t}'}$$
$$R_{\max}(t) = R_{\max,\text{CDM}}(t_f) + \int_{t_f}^t dt' \frac{dR_{\max,\text{CDM}}(t')}{dt'} + \int_{t_f}^t \frac{dt'}{t_c(t')} \frac{dR_{\max,\text{Model}}(\tilde{t}')}{d\tilde{t}'}$$

Accretion in CDM



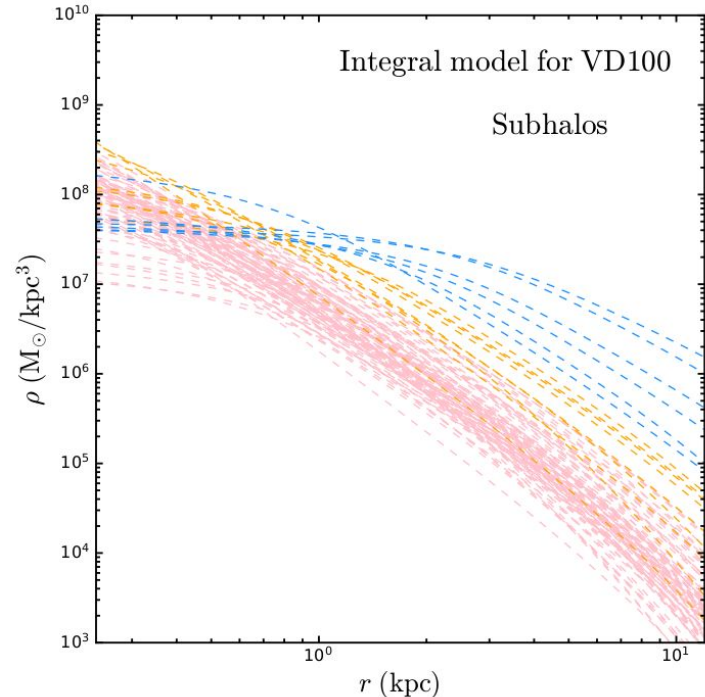
Gravothermal evolution

Apply to a population of halos



Very good
agreement

Enables
probing very
inner region
below the
resolution in
simulations



Paper in preparation

More analytic kernels?

The idea of integral model could be more generic:

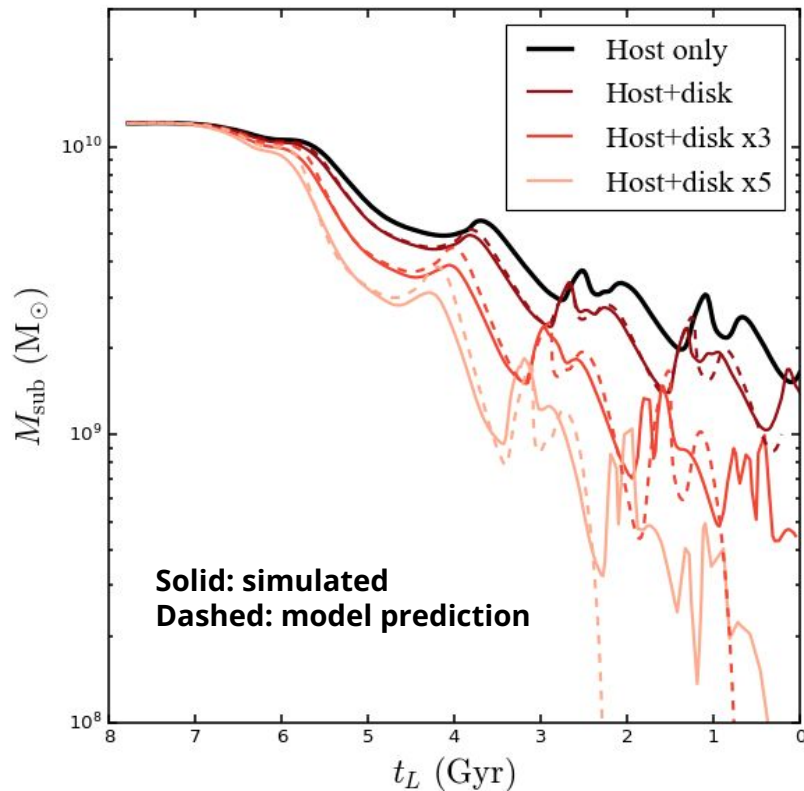
Theoretical modeling on top of accurate CDM results

A perturbative analysis for the tidal radius shows

$$\frac{d\delta M_{\text{sub}}}{dt} = -\frac{M_{\text{disk}}(d, t)}{M_{\text{host}}(d, t)} \frac{dM_{\text{sub}}}{dt}$$

An integral disk model

Paper in preparation



Thanks for your attention!